Theory of Thin Airfoils in Magnetoaerodynamics

LAZAR DRAGOS*
Bucharest University, Bucharest, Rumania

This paper examines the motion of a compressible fluid with arbitrary finite electrical conductivity in the presence of a thin airfoil. At infinity, in the undisturbed stream, the magnetic fluid is assumed to be orthogonal to the direction of the fluid stream. The motion is irrotational. The velocity field and the magnetic field have an irrotational part, which coincides to that of the classical aerodynamics, and a rotational part, which is due to the field-motion interaction and which vanishes for $A \to \infty$. The general solution depends on five arbitrary functions. The boundary conditions determine the relations between these functions as well as the boundary problem for determining one of them. This one reduces to a Fredholm-type integral equation. Surface currents are not possible. The solution valid for arbitrary Mach number M coincides with the solution of the case of incompressible fluid for $M = 0$.

Introduction

THE motion of a compressible, conducting fluid in the presence of a thin airfoil was considered by McCune and Resler.[1] Important results of this problem were also obtained by other authors.[2] In all of the cases, however, the perfectly conducting fluid was considered ($R_M = \infty$). The method of investigation is that initiated by Sears and Resler.[3]

In the present paper, the same problem dealt with in Ref. 1 will be treated for the case in which the fluid has arbitrary electrical conductivity σ (arbitrary R_M). The case (very important in practice) of the undisturbed stream where the direction of the magnetic field and that of the stream are perpendicular (crossed fields) will be here considered. The case in which the magnetic field has the same direction as the stream velocity (aligned fields) was considered in Ref. 4. The method of approaching the two cases is a unitary one. It should also be mentioned that the motion of an incompressible fluid with arbitrary conductivity in the presence of a thin airfoil was considered in Refs. 5–8.

1. Motion Equations

The motion equations of an inviscid, compressible, and conducting fluid in electromagnetic field are

$$(d\rho/dt) + \rho \operatorname{div}\mathbf{q} = 0 \tag{1.1}$$

$$\rho(d\mathbf{q}/dt) + \operatorname{grad}p = \mu_e[\mathbf{j}\cdot\mathbf{H}] \tag{1.2}$$

where the notation

$$d/dt = \partial/\partial t + (\mathbf{q}\cdot\operatorname{grad}) \tag{1.3}$$

was used.

Besides Eqs. (1.1) and (1.2), the equation of the electromagnetic field, which we write in electromagnetic units,

$$\operatorname{rot}\mathbf{H} = 4\pi\mathbf{j} \qquad \operatorname{div}\mathbf{H} = 0 \tag{1.4}$$

$$\operatorname{rot}\mathbf{E} = -\mu_e(\partial\mathbf{H}/\partial t) \qquad \operatorname{div}\mathbf{E} = 0 \tag{1.5}$$

as well as Ohm's law

$$\mathbf{j} = \sigma(\mathbf{E} + \mu_e[\mathbf{j}\cdot\mathbf{H}]) \tag{1.6}$$

must be also considered.

Here σ (the electrical conductivity) and μ_e (the magnetic permeability) are assumed to be constants characterizing the medium.

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* Assistant Professor, Faculty of Mathematics and Mechanics.

We assume that the fluid medium is characterized by the thermodynamic law

$$p = \rho RT \tag{1.7}$$

In such a case, if we denote by w the specific internal energy and by h the specific enthalpy, we have

$$w = C_vT \qquad h = w + (p/\rho) = C_pT \tag{1.8}$$

The energy equation, under the assumption that the motion takes place without heat transfer,[9, 4]

$$\rho\frac{dh}{dt} = \frac{dp}{dt} + \frac{1}{\sigma}\left(\frac{1}{4\pi}\right)^2 \operatorname{rot}\mathbf{H}\cdot\operatorname{rot}\mathbf{H} \tag{1.9}$$

We introduce the dimensionless independent variables

$$x' = x/L \qquad y' = y/L \qquad t' = tU/L \tag{1.10}$$

L being a characteristic length still undetermined and U the undisturbed stream velocity. We also introduce the dimensionless dynamic variables

$$\begin{gathered} \mathbf{q}' = \mathbf{q}/U \qquad \rho' = \rho/\rho_\infty \qquad h' = h/h_\infty \\ p' = (p - p_\infty)/\rho_\infty U^2 \qquad \mathbf{H}' = \mathbf{H}/H_\infty \\ \mathbf{E}' = \mathbf{E}/\mu_e H_\infty U \qquad \mathbf{j}' = \mathbf{j}/\sigma\mu_e U H_\infty \end{gathered} \tag{1.11}$$

With the new variables, which are written unstressed, the system of equations which yields the fluid motion is

$$\begin{gathered} (d\rho/dt) + \rho\operatorname{div}\mathbf{q} = 0 \\ \rho(d\mathbf{q}/dt) + \operatorname{grad}p = (1/A^2)[\operatorname{rot}\mathbf{H}\cdot\mathbf{H}] \end{gathered} \tag{1.12}$$

$$\begin{gathered} \operatorname{rot}\mathbf{H} = R_M\mathbf{j} \qquad \operatorname{div}\mathbf{H} = 0 \\ \operatorname{rot}\mathbf{E} = -(\partial\mathbf{H}/\partial t) \qquad \operatorname{div}\mathbf{E} = 0 \end{gathered} \tag{1.13}$$

$$\mathbf{j} = \mathbf{E} + [\mathbf{q}\cdot\mathbf{H}] \tag{1.14}$$

$$\rho\frac{dh}{dt} = (\gamma - 1)M^2\frac{dp}{dt} + (\gamma - 1)M^2\frac{1}{A^2}\cdot\frac{1}{R_M}\operatorname{rot}\mathbf{H}\cdot\operatorname{rot}\mathbf{H} \tag{1.15}$$

$$\gamma M^2 p = \rho h - 1 \tag{1.16}$$

The notations

$$\begin{gathered} R_M = LU/\nu_H = 4\pi\sigma\mu_e LU \\ \gamma = c_p/c_v, \qquad M = U/a_\infty \qquad A = U/V_A \end{gathered} \tag{1.17}$$

were introduced, where C_p and C_v are the gas specific heats under constant pressure and constant volume, respectively,

a_∞ is the sound velocity in the undisturbed stream, R_M is the magnetic Reynolds number, and V_A is the Alfvén velocity:

$$V_A = H_\infty[\mu_e/4\pi\rho_\infty]^{1/2} \tag{1.18}$$

2. Plane Steady Motion: Linearized Theory

As basic motion, we shall consider the undisturbed stream motion, which will be assumed as being uniform, having the velocity U. The Ox axis is taken upon the direction of the velocity U. The conducting fluid is in a magnetic field of intensity H_∞, the direction of which is perpendicular to the direction of U. The Oy axis is taken upon the direction of the magnetic field.

The fluid motion takes place in the presence of a thin airfoil having the equation

$$y = Y(x) \qquad -(c/L) \leqslant x \leqslant (c/L) \tag{2.1}$$

which will disturb both the velocity field and the magnetic field. Thus, taking into consideration the basic motion, the dynamic variables become

$$\begin{aligned} \mathbf{q} &= \mathbf{e}_1 + \mathbf{v} \qquad & \rho &= 1 + \rho_1 \\ \mathbf{H} &= \mathbf{e}_2 + \mathbf{h} \qquad & \mathbf{E} &= \mathbf{E}_\infty + \mathbf{E}_1 \end{aligned} \tag{2.2}$$

which must satisfy the system of Eqs. (1.12–1.16). Here $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ denote the unit vectors of the Ox, Oy, and Oz axes, respectively. If the motion is assumed to be steady, then, neglecting the second-order small magnitudes, we have

$$d/dt = (\mathbf{e}_1 + \mathbf{v}) \text{ grad} = \partial/\partial x \tag{2.3}$$

From (1.15) and (1.16) we deduce $M^2(\partial p/\partial x) = \partial\rho_1/\partial x$. Thus, we obtain the following system:

$$M^2 \frac{\partial p}{\partial x} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.4}$$

$$\frac{\partial p}{\partial x} + \frac{\partial u}{\partial x} = \frac{1}{A^2}\left(\frac{\partial h_x}{\partial y} - \frac{\partial h_y}{\partial x}\right) \tag{2.5}$$

$$\frac{\partial v}{\partial x} + \frac{\partial p}{\partial y} = 0 \tag{2.6}$$

$$\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} = R_M(h_y + u) \tag{2.7}$$

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} = 0 \tag{2.8}$$

The consideration presented in Ref. 4 shows that in this case $\mathbf{E}_\infty = -\mathbf{e}_3 E_1' = 0$. It is seen that perturbations of the electrical field do not appear, a conclusion that is to be expected since the magnetic field is stationary, and there are no electrical sources in the region occupied by the field.

Using the notations

$$\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} \qquad \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{2.9}$$

from (2.5) and (2.6) we obtain

$$\frac{\partial \omega}{\partial x} = \frac{1}{A^2}\frac{\partial \xi}{\partial y} \tag{2.10}$$

From relations (2.2) and Eq. (2.7), we have

$$\lim_{|d|\to\infty}(\mathbf{v},\mathbf{h},p,\xi) = 0 \qquad d^2 = x^2 + y^2 \tag{2.11}$$

Using now Eqs. (2.4, 2.7, and 2.8), we get

$$\frac{A^2(M^2-1)}{R_M}\frac{\partial^5\xi}{\partial x^5} + \frac{A^2(M^2-2)}{R_M}\frac{\partial^5\xi}{\partial x^3\partial y^2} - \frac{A^2}{R_M}\frac{\partial^5\xi}{\partial x\,\partial y^4} = (A^2M^2 - A^2 - M^2)\frac{\partial^4\xi}{\partial x^4} - (A^2+M^2-1)\frac{\partial^4\xi}{\partial x^2\partial y^2} + \frac{\partial^4\xi}{\partial y^4} = 0 \tag{2.12}$$

which may be also written as

$$\frac{A^2}{R_M}\frac{\partial}{\partial x}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\beta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\xi - (A^2M^2 - A^2 - M^2)\frac{\partial^4\xi}{\partial x^4} + (A^2+M^2-1)\frac{\partial^4\xi}{\partial x^2\partial y^2} - \frac{\partial^4\xi}{\partial y^4} = 0 \tag{2.13}$$

We have used the classical notation $\beta^2 = 1 - M^2$, β being a real number for subsonic motions and an imaginary number of supersonic motions. This form of Eq. (2.13) determines the type of problems appearing in such a case. The high-order derivatives are written as a product of three operators, i.e., the operator $\partial/\partial x$, the operator Δ, and the operator T, which for subsonic motions is the Prandtl-Glauert operator and for supersonic motion is the Ackeret operator. Hence, it is seen that in this case only the sonic motions appear as a singular case that separates two regimes of motion. The situation is similar to that of the motion of the nonconducting fluid.

It is worth mentioning the difference existing between the general case treated here (R_M being finite and arbitrary) and the case of the perfectly conducting fluid.[1] For the latter case, Eq. (2.12), in which we set $R_M \to \infty$, assumes the form

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{R^2}\frac{\partial^2}{\partial y^2}\right)\left(\frac{B}{R^2}\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\xi = 0 \tag{2.14}$$

The notations

$$\begin{aligned} 2R^2 &= A^2 + M^2 - 1 + [(A^2+M^2-1)^2 + 4(A^2+M^2-A^2M^2)]^{1/2} = A^2 + M^2 - 1 + [1 + 2(A^2+M^2) + (A^2-M^2)^2]^{1/2} \\ B &= A^2 + M^2 - A^2M^2 \end{aligned} \tag{2.15}$$

were used. It is seen that R^2 is positive throughout the plane (A,M). Accordingly, the first operator in (2.14) is hyperbolic, and the second one is elliptical if A and M are in the region $B > 0$ and hyperbolic in the region $B < 0$. The critical case that separates the two motion regimes is that corresponding to $U^2 = a_\infty^2 + V_A^2$.

Finally, it is to be noticed that, in the general case (R_M being finite), the type of equation in ξ [Eq. (2.12)] does not depend on the orientation of the magnetic field in the undisturbed stream, whereas in the case of the perfectly conducting fluid, the type of equation depends on the orientation of the magnetic field.

We mention also that, in the case of incompressible fluid $M = 0$, Eq. (2.12) takes the form obtained by Sears and Resler[3]:

$$\frac{1}{R_M}\frac{\partial}{\partial x}\Delta\xi = \frac{\partial^2\xi}{\partial x^2} - \frac{1}{A^2}\frac{\partial^2\xi}{\partial y^2} \tag{2.16}$$

3. Representation of the General Solution of Eq. (2.12) (R_M Being Arbitrary)

We assume that the function $\xi(x,y)$ satisfies the Dirichlet conditions relative to the variable x. Taking into consideration condition (2.11), we have

$$\int_{-\infty}^{+\infty}\xi(x,y)dx = \text{convergent}$$

In this case, introducing the Fourier transform

$$\Omega(\lambda,y) = \left[\frac{1}{2\pi}\right]^{1/2}\int_{-\infty}^{+\infty}\xi(x,y)e^{i\lambda x}\,dx \tag{3.1}$$

and taking into consideration the cancellation of disturbances at infinity (in the undisturbed stream), from Eq. (2.12) we obtain the corresponding equation for $\Omega(\lambda,y)$:

$$a\frac{d^4\Omega}{dy^4} + \lambda^2 b\frac{d^2\Omega}{dy^2} + \lambda^4 c = 0 \tag{3.2}$$

where

$$a = 1 - \frac{i\lambda A^2}{R_M} \qquad b = A^2 + M^2 - 1 - \frac{A^2(M^2-2)}{R_M}i\lambda$$

$$c = A^2M^2 - A^2 - \mathrm{M} + \frac{A^2(M^2-1)}{R_M}i\lambda \tag{3.3}$$

The characteristic equation corresponding to Eq. (3.2) is

$$E(\alpha) = a\alpha^4 + \lambda^2 b\alpha^2 + \lambda^4 c = 0 \tag{3.4}$$

It has the roots

$$\begin{aligned} r_\pm(i\lambda) &= \mp|\lambda|\left[\frac{-b + (b^2 - 4ac)^{1/2}}{2a}\right]^{1/2} \\ s_\pm(i\lambda) &= \mp|\lambda|\left[\frac{-b - (b^2 - 4ac)^{1/2}}{2a}\right]^{1/2} \end{aligned} \tag{3.5}$$

We observe that the roots $\Re_e r_\pm$, $\Re_e s_\pm$ depend on $|\lambda|$ since one may write

$$\begin{aligned} \Re_e r_\pm(i\lambda) &= \tfrac{1}{2}[r_\pm(i\lambda) + r_\pm(-i\lambda)] = \mp|\lambda|R(|\lambda|) \\ \Re_e s_\pm(i\lambda) &= \tfrac{1}{2}[s_\pm(i\lambda) + s_\pm(-i\lambda)] = \mp|\lambda|S(|\lambda|) \end{aligned} \tag{3.6}$$

In addition, $RS \neq 0$.†

With this observation and considering that the roots $r_\pm$, $s_\pm$ depend only on $|\lambda|$, the damping condition imposed by (2.11) may be used for the function $\Omega(\lambda,y)$, the Fourier transform of $\xi(x,y)$. Thus, we obtain the following general solution of Eq. (3.2):

$$\Omega(\lambda,\pm y) = C_\pm(\lambda)e^{r_\pm y} + D_\pm(\lambda)e^{s_\pm y} \tag{3.7}$$

The plus sign indicates the solution valid in the half-plane $y > 0$ and the minus sign the solution valid for $y < 0$. The solution (3.7) may be also extended on the Ox axis.

By an inversion of the Fourier integral (3.1), we have

$$\xi(x,\pm y) = \frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty}[C_\pm(\lambda)e^{r_\pm y} + D_\pm(\lambda)e^{s_\pm y}]e^{-i\lambda x}\,d\lambda \tag{3.8}$$

The functions $C_\pm(\lambda)$ and $D_\pm(\lambda)$ are to be determined subsequently. In the following considerations we shall use the notations

$$\begin{aligned} \xi_1(x,\pm y) &= \frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty} C_\pm(\lambda)e^{r_\pm y}\,e^{-i\lambda x}\,d\lambda \\ \xi_2(x,\pm y) &= \frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty} \mathrm{D}(\lambda)e^{s_\pm y}\,e^{-i\lambda x}\,d\lambda \end{aligned} \tag{3.9}$$

From (2.10) we obtain readily

$$\omega(x,\pm y) = \frac{1}{A^2[2\pi]^{1/2}}\int_{-\infty}^{+\infty}\left[C_\pm(\lambda)\frac{ir_\pm}{\lambda}e^{r_\pm y} + D_\pm(\lambda)\frac{is_\pm}{\lambda}e^{s_\pm y}\right]e^{-i\lambda x}\,d\lambda = \omega_1 + \omega_2 \tag{3.10}$$

using the same notations as in (3.9).

† Since $R(|\lambda|) = R(0) + (|\lambda|/1!)R'(0) + \ldots$, the cancellation of $R(|\lambda|)$ would imply $R(0) = R'(0) = \ldots = 0$. A simple calculation taking into consideration (2.15) gives, however,

$$[2]^{1/2}R(0) = \{A^2 + M^2 - 1 + [1 + 2(A^2 + M^2) + (A^2 - M^2)^2]^{1/2}\}^{1/2}$$

which obviously cannot be zero.

4. Determination of the Velocity Field

From Eqs. (2.4) and (2.5) and considering the notation (2.9), we have

$$(1 - M^2)(\partial u/\partial x) + (\partial v/\partial y) = (M^2/A^2)\xi \tag{4.1}$$

For determining the velocity field, we also obtain from (2.9)

$$(\partial v/\partial x) - (\partial u/\partial y) = \omega \tag{4.2}$$

The general solution of system (4.1) and (4.2) is obtained by taking the general solution of the homogeneous system to which we add a particular solution of the nonhomogeneous system. The general solution of the homogeneous system is

$$u = \partial\varphi/\partial x \qquad v = \partial\varphi/\partial y \tag{4.3}$$

the function φ satisfying the equation

$$(1 - M^2)(\partial^2\varphi/\partial x^2) + (\partial^2\varphi/\partial y^2) = 0 \tag{4.4}$$

The particular solution of the nonhomogeneous system is sought under the form $u_P = u_{P_1} + u_{P_2}$, $v_P = v_{P_1} + v_{P_2}$, where

$$\begin{aligned} u_{P_1} &= \frac{1}{A^2[2\pi]^{1/2}}\int_{-\infty}^{+\infty} C_\pm(\lambda)\varphi_{P_1}(x,y,\lambda)d\lambda \\ u_{P_2} &= \frac{1}{A^2[2\pi]^{1/2}}\int_{-\infty}^{+\infty} D_\pm(\lambda)\varphi_{P_2}(x,y,\lambda)d\lambda \\ v_{P_1} &= \frac{1}{A^2[2\pi]^{1/2}}\int_{-\infty}^{+\infty} C_\pm(\lambda)\psi_{P_1}(x,y,\lambda)d\lambda \\ v_{P_1} &= \frac{1}{A^2[2\pi]^{1/2}}\int_{-\infty}^{+\infty} D_\pm(\lambda)\psi_{P_1}(x,y,\lambda)d\lambda \end{aligned} \tag{4.5}$$

The nonhomogeneous system is satisfied if we take

$$\begin{aligned} \beta^2\frac{\partial\varphi_{P_1}}{\partial x} + \frac{\partial\psi_{P_1}}{\partial y} &= M^2e^{r_\pm y}\,e^{-i\lambda x} \\ \frac{\partial\psi_{P_1}}{\partial x} - \frac{\partial\varphi_{P_1}}{\partial y} &= \frac{ir_\pm}{\lambda}e^{r_\pm y}\,e^{-i\lambda x} \end{aligned} \tag{4.6}$$

$$\begin{aligned} \beta^2\frac{\partial\varphi_{P_2}}{\partial x} + \frac{\partial\psi_{P_2}}{\partial y} &= M^2e^{s_\pm y}e^{-i\lambda x} \\ \frac{\partial\psi_{P_2}}{\partial x} - \frac{\partial\varphi_{F_2}}{\partial y} &= \frac{is_\pm}{\lambda}e^{s_\pm y}\,e^{-i\lambda x} \end{aligned} \tag{4.7}$$

For system (4.6) the solution is looked for under the form

$$\varphi_{P_1} = C_1e^{r_\pm y}\,e^{-i\lambda x} \qquad \psi_{P_1} = C_2e^{r_\pm y}\,e^{-i\lambda x}$$

C_1 and C_2 being constants with respect to the integration variables x,y, and correspondingly for system (4.7). Finally, we have

$$\begin{aligned} u_P(x,\pm y) &= \frac{1}{A^2[2\pi]^{1/2}}\int_{-\infty}^{+\infty}\left[\frac{r^2 + M^2\lambda^2}{i\lambda(r^2 - \beta^2\lambda^2)}C_\pm(\lambda)\,e^{r_\pm y} + \frac{s^2 + M^2\lambda^2}{i\lambda(s^2 - \beta^2\lambda^2)}D_\pm(\lambda)e^{s_\pm y}\right]e^{-i\lambda x}\,d\lambda \\ v_P(x,\pm y) &= \frac{1}{A^2[2\pi]^{1/2}}\int_{-\infty}^{+\infty}\left[\frac{r_\pm}{r^2 - \beta^2\lambda^2}C_\pm(\lambda)e^{r_\pm y} + \frac{s_\pm}{s^2 - \beta^2\lambda^2}D_\pm(\lambda)e^{s_\pm y}\right]e^{-i\lambda x}\,d\lambda \\ r^2 &= r_\pm{}^2 \qquad s^2 = s_\pm{}^2 \end{aligned} \tag{4.8}$$

Accordingly, the velocity field has the following general representation:

$$u(x,\pm y) = u_P(x,\pm y) + (\partial\varphi/\partial x) \qquad v(x,\pm y) = v_P(x,\pm y) + (\partial\varphi/\partial y) \tag{4.9}$$

The function φ satisfies Eq. (4.4), which, in the case of subsonic motions, coincides with the Glauert and Prandtl equation, whereas, in the case of supersonic motions, it coincides with Ackeret's equation.

Taking into consideration Eq. (3.4) and performing a simple calculation, we obtain

$$(r^2 - \beta^2\lambda^2)(s^2 - \beta^2\lambda^2) = \frac{-M^2\lambda^4}{1 - (i\lambda A^2/R_M)} \tag{4.10}$$

Hence, the representation (4.8) is valid in the entire (A,M) plane. It is not valid for $M = 0$, that is, in the case of the incompressible fluid. The explanation of this fact is that, in the case of the incompressible fluid, the equation in ξ [Eq. (2.16)] is of low order and its characteristic equation (3.4) has only two roots. We mention also that in this case the roots obtained from (3.5) coincide with those found previously for the incompressible fluid.[7]

Consequently, the velocity has two components, i.e., a potential component, which satisfies the same equation as in the case of the nonconducting fluid, and another one (u_P,v_P) due to the interaction between the motion and the magnetic field. By setting $A \to \infty$, we obtain the motion of the nonconducting fluid. [The motion equation is (1.12), in which the term representing the action of the magnetic field vanishes.]

Taking into consideration that from (3.5) we have

$$\lim_{A\to\infty} r_\pm \neq \infty \qquad \lim_{A\to\infty} s_\pm \neq \infty$$

from (3.10) we obtain

$$\lim_{A\to\infty} \omega = 0$$

that is, the motion is irrotational. System (4.1) and (4.2) reduces to the homogeneous system whose solution is (4.3) and (4.4); that is, we obtain the results given by the classical aerodynamics

$$\lim_{A\to\infty} (u_P,v_P) = 0$$

5. Determination of the Magnetic Field

For determining the magnetic field, we have the following equations:

$$\mathrm{div}\,\mathbf{h} = 0 \qquad \mathrm{rot}\,\mathbf{h} = \xi\mathbf{e}_3 \tag{5.1}$$

We are confronted, hence, with a Poincaré-Steelhoff problem. The solution may be simply determined by following the same procedure as in the case of the velocity field. We have

$$h_x(x,\pm y) = \frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty}\left[\frac{r_\pm}{\lambda^2 - r^2}\,C_\pm(\lambda)e^{r_\pm y} + \frac{s_\pm}{\lambda^2 - s^2}\,D_\pm(\lambda)e^{s_\pm y}\right]e^{-i\lambda x}\,d\lambda + \frac{\partial\psi}{\partial x} = h_{xR} + \frac{\partial\psi}{\partial x} \tag{5.2}$$

$$h_y(x,\pm y) = \frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty}\left[\frac{i\lambda}{\lambda^2 - r^2}\,C_\pm(\lambda)e^{r_\pm y} + \frac{i\lambda}{\lambda^2 - s^2}\,D_\pm(\lambda)e^{s_\pm y}\right]e^{-i\lambda x}\,d\lambda + \frac{\partial\psi}{\partial y} = h_{yR} + \frac{\partial\psi}{\partial y}$$

Accordingly, the magnetic field may be represented as

$$\mathbf{h} = \mathbf{h}_R + \mathrm{grad}\,\psi \tag{5.3}$$

The function ψ, which determines the irrotational part of the magnetic field, satisfies the equations

$$\Delta\psi = 0 \qquad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{5.4}$$

Similarly, using Eq. (3.4), we deduce

$$(\lambda^2 - r^2)(\lambda^2 - s^2) = \frac{A^2M^2\lambda^4}{1 - (i\lambda A^2/R_M)} \tag{5.5}$$

It is seen that the representation (5.2) is valid in the entire (A, M) plane.

6. Useful Relations

From the expressions of the functions $\xi(x,\pm y)$, $u_p(x,\pm y)$, $v_p(x,\pm y)$, $h_{yR}(x,\pm y)$, $h_{xR}(x,\pm y)$ and by an inversion of the Fourier transforms, we obtain

$$E_1 = C_\pm(\lambda)e^{r_\pm y} + D_\pm(\lambda)e^{s_\pm y} = \frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty}\xi(x,\pm y)e^{i\lambda x}\,dx \tag{6.1}$$

$$E_2 = \frac{1}{A^2}\left[\frac{ir_\pm}{\lambda}\,C_\pm(\lambda)e^{r_\pm y} + \frac{is_\pm}{\lambda}\,D_\pm(\lambda)e^{s_\pm y}\right] = \frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty}\omega(x,\pm y)e^{i\lambda x}\,dx \tag{6.2}$$

$$E_3 = \frac{1}{A^2}\left[\frac{r^2 + M^2\lambda^2}{i\lambda(r^2 - \beta^2\lambda^2)}\,C_\pm e^{r_\pm y} + \frac{s^2 + M^2\lambda^2}{i\lambda(s^2 - \beta^2\lambda^2)}\,D_\pm e^{s_\pm y}\right] = \frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty}u_P(x,\pm y)e^{i\lambda x}\,dx \tag{6.3}$$

$$E_4 = \frac{1}{A^2}\left[\frac{r_\pm}{r^2 - \beta^2\lambda^2}\,C_\pm e^{r_\pm y} + \frac{s_\pm}{s^2 - \beta^2\lambda^2}\,D_\pm e^{s_\pm y}\right] = \frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty}v_P(x,\pm y)e^{i\lambda x}\,dx \tag{6.4}$$

$$E_5 = \frac{r_\pm}{r^2 - \lambda^2}\,C_\pm e^{r_\pm y} + \frac{s_\pm}{s^2 - \lambda^2}\,D_\pm e^{s_\pm y} = -\frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty}h_{xR}(x,\pm y)e^{i\lambda x}\,dx \tag{6.5}$$

$$E_6 = \frac{i\lambda}{r^2 - \lambda^2}\,C_\pm e^{r_\pm y} + \frac{i\lambda}{s^2 - \lambda^2}\,D_\pm e^{s_\pm y} = -\frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty}h_{yR}(x,\pm y)e^{i\lambda x}\,dx \tag{6.6}$$

The foregoing functions may be continuously extended on the Ox axis. Hence,

$$L_1 = C_\pm(\lambda) + D_\pm(\lambda) = \frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty}\xi(x,\pm 0)e^{i\lambda x}\,dx \tag{6.1'}$$

$$L_4 = \frac{1}{A^2}\left[\frac{r_\pm}{r^2 - \beta^2\lambda^2}\,C_\pm + \frac{s_\pm}{s^2 - \beta^2\lambda^2}\,D_\pm\right] = \frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty}v_P(x,\pm 0)e^{i\lambda x}\,dx \tag{6.4'}$$

$$L_6 = \frac{i\lambda}{\lambda^2 - r^2}\,C_\pm + \frac{i\lambda}{\lambda^2 - s^2}\,D_\pm = \frac{1}{[2\pi]^{1/2}}\int_{-\infty}^{+\infty}h_{yR}(x,\pm 0)e^{i\lambda x}\,dx \tag{6.6'}$$

A direct calculation leads to the relation

$$A^2E_3(r^2 - \beta^2\lambda^2)(s^2 - \beta^2\lambda^2) + E_4(r^2 - \lambda^2)(s^2 - \lambda^2) = M^2\lambda^2(r_\pm C_\pm e^{r_\pm y} + s_\pm D_\pm e^{s_\pm y}) \tag{6.7}$$

Taking into consideration (4.10) and (5.5), we deduce

$$i\lambda E_3 - i\lambda E_4 = \left(\frac{1}{i\lambda A^2} - \frac{1}{R_M}\right)(r_\pm C_\pm e^{r_\pm y} + s_\pm D_\pm e^{s_\pm y}) \tag{6.8}$$

Further, taking into account Eqs. (6.1–6.4), we obtain

$$\frac{\partial v_P}{\partial x} - \frac{\partial h_{xR}}{\partial x} = \omega + \frac{1}{R_M}\frac{\partial \xi}{\partial y} \tag{6.9}$$

Observing that

$$\omega = \frac{\partial v_P}{\partial x} - \frac{\partial u_P}{\partial y} \qquad \frac{\partial h_{xR}}{\partial x} + \frac{\partial h_{yR}}{\partial y} = 0 \tag{6.10}$$

relation (6.9) becomes

$$\frac{1}{R_M}\frac{\partial \xi}{\partial y} = \frac{\partial h_{yR}}{\partial y} + \frac{\partial u_P}{\partial y} \tag{6.11}$$

On comparing relation (6.11) with Ohm's law [Eq. (2.7)], in which the representations (4.9) and (5.3) were used, we obtain

$$\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial x \partial y} = 0 \tag{6.12}$$

7. Boundary Conditions

We assume that the airfoil is infinitely thin and of small camber. In this case, using Eqs. (2.1), the condition that the fluid velocity be tangent to the airfoil is

$$v_P(x,\pm 0) + \left.\frac{\partial \varphi}{\partial y}\right|_{y=\pm 0} = Y'(x) \qquad -1 \le x \le 1 \tag{7.1}$$

For simplifying the writing, the origin of the reference system was taken such as to coincide with the middle of the airfoil, and the characteristic length $2L$, so far indetermined, is chosen equal to the length of the profile chord.

In the exterior of the profile, on the Ox axis, the velocity component v is continuous, as it results from a known theorem.[4] Hence, at all points on the Ox axis we have

$$v_P(x,+0) - v_P(x,-0) = \left.\frac{\partial \varphi}{\partial y}\right|_{-0} - \left.\frac{\partial \varphi}{\partial y}\right|_{+0} \tag{7.2}$$

Introducing the corresponding notations, relation (7.2) becomes

$$[v_P] = -[\partial \varphi/\partial y] \rightarrow [v] = 0 \tag{7.3}$$

The magnetic field components are also continuous (surface currents do not exist), such that, using the same notations as in (7.3), we have

$$[h_x] = 0 \qquad [h_y] = 0 \tag{7.4}$$

From (2.7), taking into consideration (7.4), we obtain $[\xi] = R_M[u]$. Deriving (2.5) with respect to the variable y and taking into account the relation $[\partial p/\partial y] = 0$, which results from (2.6), we get

$$\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial y}\right] = -\frac{1}{A^2}\left[\frac{\partial \xi}{\partial y}\right] \tag{7.5}$$

Deriving (2.7) with respect to the variables y and taking into account the relation $[\partial h_y/\partial y] = 0$, which results from (2.8) and (7.4), we obtain

$$\frac{1}{R_M}\left[\frac{\partial \xi}{\partial y}\right] = \left[\frac{\partial u}{\partial y}\right] \tag{7.6}$$

From relations (7.5) and (7.6) we have

$$\left[\frac{\partial u}{\partial y}\right] = C \exp\left(-\frac{R_M}{A^2}x\right) \tag{7.7}$$

Considering now the damping condition (2.11), we have‡

$$[\partial u/\partial y] = 0 \tag{7.8}$$

There results $[\omega] = 0$, and, accordingly, making use of (3.16) and (3.20), we get

$$r_+(C_+ + C_-) + s_+(D_+ + D_-) = 0 \tag{7.9}$$

On the other hand, taking into consideration the representation (5.3), we obtain from (7.4) $[h_{xR}] = -[\partial\psi/\partial x] = 0$. From (6.5) we have

$$\frac{r_+}{\lambda^2 - r^2}(C_+ + C_-) + \frac{s_+}{\lambda^2 - s^2}(D_+ + D_-) = 0 \tag{7.10}$$

By comparing (7.9) and (7.10) we have

$$C_+(\lambda) = -C_-(\lambda) \qquad D_+(\lambda) = -D_-(\lambda) \tag{7.11}$$

relations which determine completely the symmetry of the problem.

From Eqs. (6.1–6.6) there results

$$\begin{aligned} \omega(x,+y) &= \omega(x,-y) & \xi(x,+y) &= -\xi(x,-y) \\ v_P(x,+y) &= v_P(x,-y) & u_P(x,+y) &= -u_P(x,-y) \\ h_{xR}(x,+y) &= h_{xR}(x,-y) & h_{yR}(x,+y) &= -h_{yR}(x,-y) \end{aligned} \tag{7.12}$$

From the symmetry of $v_P(x,\pm y)$ and $h_{xR}(x,\pm y)$, using (7.3) and (7.4), we obtain

$$\left.\frac{\partial \varphi}{\partial y}\right|_{-0} = \left.\frac{\partial \varphi}{\partial y}\right|_{+0} \qquad \left.\frac{\partial \psi}{\partial x}\right|_{-0} = \left.\frac{\partial \psi}{\partial x}\right|_{+0} \tag{7.13}$$

Using these relations in condition (6.12), we readily obtain

$$\left.\frac{\partial \psi}{\partial y}\right|_{-0} = -\left.\frac{\partial \psi}{\partial y}\right|_{+0} \tag{7.14}$$

These relations determine completely the symmetry of the problem. Employing (7.4) and (7.14) and the antisymmetry of h_{yR}, we obtain the relations

$$h_{yR}(x,\pm 0) = -(\partial\psi/\partial y)|_{\pm 0} \tag{7.15}$$

On the other hand, the partial derivations of the harmonic function ψ are continuous in the exterior of the profile. It results that

$$(\partial\psi/\partial y)|_{\pm 0} = 0 \qquad (-\infty < x < -1, 1 < x < \infty) \tag{7.16}$$

Further, the notations

$$I(\lambda) = \frac{-\dfrac{r_\pm}{s_\pm}\left(\dfrac{1}{R_M} - \dfrac{i\lambda}{\lambda^2 - r^2} - \dfrac{1}{i\lambda A^2}\cdot\dfrac{r^2 + M^2\lambda^2}{r^2 - \beta^2\lambda^2}\right)}{\dfrac{1}{R_M} - \dfrac{i\lambda}{\lambda^2 - s^2} - \dfrac{1}{i\lambda A^2}\cdot\dfrac{s^2 + M^2\lambda^2}{s^2 - \beta^2\lambda^2}} \tag{7.17}$$

will be introduced.

Taking into consideration (6.1, 6.3, 6.6, and 7.17), from (6.11) we obtain

$$D_\pm(\lambda) = I(\lambda)C_\pm(\lambda) \tag{7.18}$$

Accordingly, there results that, from the four functions that intervene in the general representation of the velocity and magnetic fields, only one [e.g., $C_+(\lambda) = C(\lambda)$] is unknown. The remaining functions are determined with the aid of relations (7.11) and (7.18).

‡ We obtain

$$u(x,+\epsilon) = u(x,0) + \int_0^{\epsilon}\frac{\partial u_+}{\partial y}\,dy,\ u(x,-\epsilon) = u(x,0) + \int_0^{-\epsilon}\frac{\partial u_-}{\partial y}\,dy$$

By substracting the two relations and applying an averaging formula, we obtain $u(x,+\epsilon) - u(x,-\epsilon) \cong [\partial u/\partial y]\epsilon$. Setting $x \rightarrow -\infty$ and considering (2.11) and (7.7), we have $C = 0$.

From relation (6.6′), taking into consideration (7.18, 7.15, and 7.16), we have

$$\left[\frac{i\lambda}{\lambda^2 - r^2} + \frac{i\lambda I(\lambda)}{\lambda^2 - s^2}\right] C(\lambda) = \frac{1}{[2\pi]^{1/2}} \int_{-1}^{+1} \frac{\partial \psi}{\partial y}\bigg|_{y=+0} e^{i\lambda x}\, dx \quad (7.19)$$

This relation will determine the function $C(\lambda)$ if the function $(\partial\psi/\partial y)|_{+0}$ is known. On the other hand, from relation (6.12) the function $\varphi(x,y)$ may be determined if the function $\psi(x,y)$ is known.

The function $\varphi(x,y)$ satisfies Eq. (4.4), and its determination will be made in compliance with Glauert and Prandtl's method[10] in the case of subsonic motions and according to Ackeret's method in the case of supersonic motions. Condition (7.1) will be used, in which $v_P(x,\pm 0)$ may be known through the determination of $C(\lambda)$.

The function $\psi(x,y)$ is, hence, the single unknown of the problem.

The boundary conditions for the magnetic field were used by assuming that surface currents do not exist. It may be easily seen that, in this case of orthogonal fields, surface currents cannot exist in the airfoil. Indeed, the existence of surface currents in the airfoil would impose that the first condition (7.4) should be written

$$h_{xR}(x,+0) - h_{xR}(x,-0) = \frac{\partial\psi}{\partial x}\bigg|_{-0} - \frac{\partial\psi}{\partial x}\bigg|_{+0} + R_M\theta(x) \quad (7.20)$$

$\theta(x)$ being the density of surface currents upon the Oz axis. As h_{xR} and $\partial\psi/\partial x$ are symmetrical, relation (7.20) implies $\theta(x) \equiv 0$.

8. Boundary Condition for the Function $\psi(x,y)$

For determining the boundary condition for the function $\psi(x,y)$, we shall use condition (7.1). For this purpose, we need a relationship between $v_P(x,\pm 0)$ and $h_{yR}(x,\pm 0)$. This one may be readily deduced from relations (6.4′) and (6.6′). Introducing the notation

$$J_\pm(\lambda) = -\left[\frac{r_\pm}{r^2 - \beta^2\lambda^2} + \frac{s_\pm I(\lambda)}{s^2 - \beta^2\lambda^2}\right] \Big/ i\lambda A^2 \left[\frac{1}{r^2 - \lambda^2} + \frac{I(\lambda)}{s^2 - \lambda^2}\right] \quad (8.1)$$

we have

$$J_+(\lambda) = -J_-(\lambda) \quad (8.2)$$

Using now relations (6.4′, 6.6′, and 7.18), we obtain

$$L_4(\lambda) = J_\pm(\lambda) L_6(\lambda) \quad (8.3)$$

Introducing the notation

$$K(x) = \frac{1}{[2\pi]^{1/2}} \int_{-\infty}^{+\infty} J_+(\lambda) e^{i\lambda x}\, d\lambda \quad (8.4)$$

and observing that, from (6.4′) and (6.6′), we have

$$v_P(x,\pm 0) = \frac{1}{[2\pi]^{1/2}} \int_{-\infty}^{+\infty} L_4(\lambda) e^{-i\lambda x}\, d\lambda$$
$$h_{yR}(x,\pm 0) = \frac{1}{[2\pi]^{1/2}} \int_{-\infty}^{+\infty} L_6(\lambda) e^{-i\lambda x}\, d\lambda \quad (8.5)$$

it results that relation (8.3) represents a relationship among the Fourier transforms of the functions $v_P(x,\pm 0)$, $K(x)$, and $h_{yR}(x,\pm 0)$. Using the convolution theorem, known in the theory of Fourier transforms, from relation (8.3) we have

$$v_P(x,\pm 0) = \pm \frac{1}{[2\pi]^{1/2}} \int_{-\infty}^{+\infty} h_{yR}(\alpha,\pm 0) K(x - \alpha)\, d\alpha \quad (8.6)$$

Here we have used relation (8.2). We shall now derive relation (8.6) with respect to x and consider condition (7.1) and relations (6.12, 7.15, and 7.16). We have

$$Y(x)'' + \frac{\partial^2\psi}{\partial y^2}\bigg|_{y=\pm 0} = \mp \frac{1}{[2\pi]^{1/2}} \int_{-1}^{+1} \frac{\partial\psi}{\partial y}\bigg|_{\substack{x=\alpha\\ y=\pm 0}} \times K'(x-\alpha)\, d\alpha \qquad -1 \leq x \leq 1 \quad (8.7)$$

This is the boundary condition for the function ψ. Certain clarifications are to be made in conjunction with the boundary condition (8.7). Partial results of this paper were included in a note published in Comptes Rendus.[11] The boundary condition was given for the function $\varphi(x,y)$, which may be readily obtained from (8.6). This formulation is, however, inconvenient in that, in the case of supersonic motions, when the equation satisfied by φ is hyperbolic, the solution hardly can be constructed. Therefore we prefer the present formulation of the boundary condition for the function ψ, which is harmonic. This problem is liable in all cases to a simpler presentation.

9. Another Form of the Boundary Condition (8.7)

The boundary condition (8.7), which is to be satisfied by the harmonic function ψ, has an integral form that meanwhile considers the values of the function ψ in the exterior of the Ox axis. It may, however, be reduced to an integro-differential equation that refers only to the values on the segment $(-1, +1)$ $y = +0$. This will be made in the following considerations.

We consider the analytic function $f(z) = \psi(x,y) + i\Psi(x,y)$ in the exterior of the segment $(-1, +1)$. Using the Cauchy-Riemann relations in $f(z)$ and $f'(z)$, we have

$$f'(z) = \frac{\partial\psi}{\partial x} - i\frac{\partial\psi}{\partial y}$$
$$f''(z) = -\frac{\partial^2\psi}{\partial y^2} - i\frac{\partial^2\psi}{\partial x\partial y} \quad (9.1)$$

Taking into consideration condition (7.16), it results that it is enough to determine the harmonic function $\partial\psi/\partial y$, which is the solution of Eq. (8.7) in the upper half-plane. In the lower half-plane it is obtained by prolonging $f'(z)$ by symmetry on the segment $(-\infty, -1)$ according to Schwartz's principle. Indeed, we have $Jmf'(z)|_{(-\infty,-1)} = 0$. At the symmetrical points $y = +0$, $y = -0$ on the segment $(-1, +1)$, the function $f'(z)$ assumes conjugate values; hence $\partial\psi/\partial y$ satisfies relation (8.7) for $y = -0$.

The harmonic function $\partial\psi/\partial y$ in the upper half-plane is now to be determined with the following boundary conditions:

$$\partial\psi/\partial y = 0 \qquad (-\infty < x < -1, 1 < x < \infty) \quad (9.2)$$

$$\frac{\partial^2\psi}{\partial y^2}\bigg|_{y=+0} = -Y''(x) - \frac{1}{[2\pi]^{1/2}} \int_{-1}^{+1} \frac{\partial\psi}{\partial y}\bigg|_{\substack{x=\alpha\\ y=+0}} \times K'(x-\alpha)\, d\alpha \qquad -1 \leq x \leq 1 \quad (9.3)$$

The solution of this problem is obtained by first solving the problem

$$\partial^2\psi/\partial x\partial y = 0 \qquad (-\infty < x < -1, 1 < x < +\infty)$$
$$\partial^2\psi/\partial y^2 = g(x) \qquad (-1 \leq x \leq 1) \quad (9.4)$$

Using the procedure given in Ref. 4 and taking into consideration (9.1), we obtain the solution

$$f''(z) = \frac{1}{\pi}[z^2 - 1]^{1/2}\int_{-1}^{+1}\frac{g(x)}{[1-x^2]^{1/2}}\frac{dx}{z-x} \tag{9.5}$$

Here we must consider that condition $f''(z)$ must be finite at infinity. This imposes

$$\int_{-1}^{+1}\frac{g(x)dx}{[1-x^2]^{1/2}} = 0 \tag{9.6}$$

In (9.5) we shall pass to limit by setting $z \to t$, t being a point on the profile. Applying Plemelj's formula, we obtain

$$f''(t) = g(t) + \frac{i}{\pi}[1-t^2]^{1/2}\int_{-1}^{+1}\frac{g(x)}{[1-x^2]^{1/2}}\frac{dx}{t-x} \qquad -1 \leq t \leq +1 \tag{9.7}$$

(At the profile, the Kutta-Jucovski condition is applicable.) The integral was taken in principal value in the Cauchy's sense. Separating the imaginary part, we have

$$-\left.\frac{\partial^2\psi}{\partial x\partial y}\right|_{\substack{x=t\\y=+0}} = \frac{1}{\pi}[1-t^2]^{1/2}\int_{-1}^{+1}\frac{g(x)}{[1-x^2]^{1/2}}\frac{dx}{t-x} \qquad -1 \leq t \leq +1 \tag{9.8}$$

Considering that

$$g(x) = -Y''(x) - \frac{1}{[2\pi]^{1/2}}\int_{-1}^{+1}\left.\frac{\partial\psi}{\partial y}\right|_{y=+0}K'(x-\alpha)d\alpha \qquad -1 \leq x \leq +1 \tag{9.9}$$

it results that (9.8) reduces to the equation

$$\left.\frac{\partial^2\psi}{\partial x\partial y}\right|_{\substack{x=t\\y=+0}} = G(t) + \int_{-1}^{+1}\left.\frac{\partial\psi}{\partial y}\right|_{\substack{x=\alpha\\y=+0}}N(t,\alpha)d\alpha \qquad -1 \leq t \leq +1 \tag{9.10}$$

The notations

$$G(t) = \frac{1}{\pi}[1-t^2]^{1/2}\int_{-1}^{+1}\frac{Y''(x)}{[1-x^2]^{1/2}}\frac{dx}{t-x} \tag{9.11}$$

$$N(t,\alpha) = \frac{1}{\pi}\cdot\frac{1}{[2\pi]^{1/2}}[1-t^2]^{1/2}\int_{-1}^{+1}\frac{K'(x-\alpha)}{[1-x^2]^{1/2}}\frac{dx}{t-x} \tag{9.12}$$

were used, the integrals being taken in principal values.

Denoting

$$\partial\psi/\partial y|_{y=+0} = \nu(x) \tag{9.13}$$

Eq. (9.10) assumes the form of an integro-differential equation

$$\nu'(t) = G(t) + \int_{-1}^{+1}\nu(\alpha)N(t,\alpha)d\alpha \qquad -1 \leq t \leq 1 \tag{9.14}$$

This is the final form of the boundary condition. It is an ordinary integro-differential equation. The methods for its examination are the classical ones. It is to be mentioned that, for determining the solution, condition (9.6), which becomes a condition for $\nu(x)$ if (9.9) is taken into account, must be considered.

By integration with respect to t, Eq. (9.14) reduces to a Fredholm-type integral equation. In the case of the incompressible fluid $M = 0$, the equation coincides to that given in Ref. 8.

This equation solves completely the problem, since, by knowing $(\partial\psi/\partial y)|_{y=+0}$ on the segment $(-1,+1)$ and taking into account condition (7.16), it results that the harmonic function ψ may be determined in the upper half-plane with the aid of a Neumann problem.

From relations (7.19) we obtain $C(\lambda)$, and from relations (7.18) and (7.11) we have $C_{\pm}(\lambda)$, $D_{\pm}(\lambda)$.

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